# Q1.d

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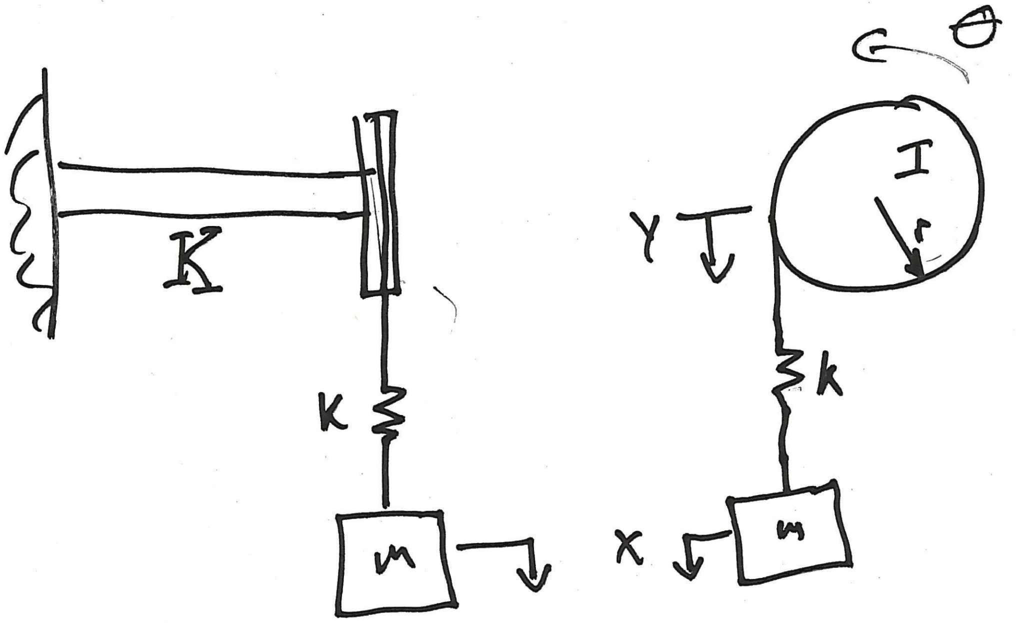
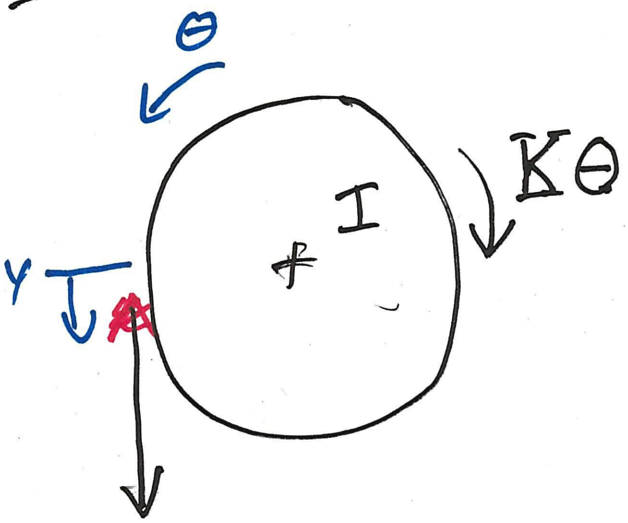



FBD θ

$y = r\theta$ (1)



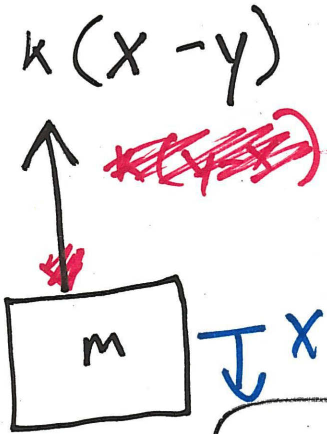

EOM θ $\quad \Sigma M_o = I\ddot{\theta}$

$-\underline{K}\theta + k(x-y)r = I\ddot{\theta}$

$-\underline{K}\theta + kr(x - r\theta) = I\ddot{\theta}$

$\boxed{(3)\ I\ddot{\theta} + \underline{K}\theta - krx + kr^2\theta = 0}$

FBD X




EOM X $\quad \Sigma F_x = m\ddot{x}$

$-k(x-y) = m\ddot{x}$

$\boxed{(2)\ m\ddot{x} + kx - kr\theta = 0}$

$$(2)\quad m\ddot{x} + kx - kr\theta = 0$$

$$(3)\quad I\ddot{\theta} + \bar{K}\theta - krx + kr^2\theta = 0$$

$$\hookrightarrow I\ddot{\theta} - krx + (kr^2 + \bar{K})\theta = 0$$

$$\begin{array}{c} (3) \\ (2) \end{array}\begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} kr^2+\bar{K} & -kr \\ -kr & k \end{bmatrix}\begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$M\ddot{z} + Kz = 0$$

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$\omega_n$ & Mode

Assume harmonic motion of amplitude ($X$ and $\Theta$), at freq $\omega$.

$$x = X\cos\omega t \qquad \theta = \Theta\cos\omega t$$

$$\ddot{x} = -\omega^2 X\cos\omega t \qquad \ddot{\theta} = -\omega^2\Theta\cos\omega t$$

$$(2) \Rightarrow -m\omega^2 X\cancel{\cos\omega t} + kX\cancel{\cos\omega t} - kr\Theta\cancel{\cos\omega t} = 0$$

$$(2b) = -m\omega^2 X + kX - kr\Theta = 0$$

$$\boxed{(2b)\quad (k - m\omega^2)X - kr\Theta = 0}$$

$$(3) \Rightarrow -I\Theta\omega^2\cancel{\cos\omega t} - krX\cancel{\cos\omega t} + (kr^2+\bar{K})\Theta\cancel{\cos\omega t} = 0$$

$$-I\Theta\omega^2 - krX + (kr^2+\bar{K})\Theta = 0$$

$$\boxed{(3b) \Rightarrow (kr^2 + \bar{K} - I\omega^2)\Theta - krX = 0}$$

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$$\begin{array}{l} (3b) \\ (2b) \end{array} \begin{bmatrix} kr^2 + \underline{K} - I\omega^2 & -kr \\ -kr & k - m\omega^2 \end{bmatrix} \begin{Bmatrix} \Theta \\ \underline{X} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$[Z]\{X\} = 0$$

$$([K] - \omega^2 [M])\{X\} = \{0\}$$

$$\lambda$$

To solve for $\omega n$

$$\det |Z| = 0$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 0$$

$$Z = \begin{bmatrix} kr^2 + \underline{K} - I\omega^2 & -kr \\ -kr & k - m\omega^2 \end{bmatrix}$$

$$\underline{Im\,\omega^4 - (Ik + kmr^2 + \underline{K}m)\omega^2 + \underline{K}k = 0}$$

$$a u^2 + b u + c = 0$$

$$u = \omega^2$$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\omega_n = \sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$a = Im$$

$$b = -Ik + kmr^2 + Km$$

$$c = Kk$$

$$\omega_{n1} = \sqrt{\frac{-b - \sqrt{b^2 - 4ac}}{2a}}$$

$$\omega_{n2} = \sqrt{\frac{-b + \sqrt{b^2 - 4ac}}{2a}}$$

lower value is $\omega_{n1}$ !!

# Solve eigenvectors for Mode shapes

$$\begin{matrix}(3b)\\(2b)\end{matrix}\begin{bmatrix} kr^2+\underline{K}-I\omega^2 & -kr \\ -kr & K-m\omega^2 \end{bmatrix}\begin{Bmatrix}H\\X\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

- To solve for mode shapes give one value unity & find other values relative to this

- ∴ if $X = 1$

using (2b)

$$-Kr\,H + (k - m\omega^2)(1) = 0$$

$$H = \frac{-m\omega^2+k}{kr}$$

$$\begin{Bmatrix}H\\X\end{Bmatrix}=\begin{Bmatrix}\frac{-m\omega^2+k}{kr}\\1\end{Bmatrix}$$

To solve for specific mode shapes input desired frequency ($\omega_{n_1}$ or $\omega_{n_2}$)

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Note: It does not matter if you use equations (3b) or (2b) w/ a unity value for $X$ or $H$

if you use (3b) w/ $X = 1$

$$(kr^2+\underline{K}-I\omega^2)\,H - kr\,(1) = 0$$

$$H = \frac{kr}{kr^2+\underline{K}-I\omega^2}$$

$$\boxed{\begin{Bmatrix}H\\X\end{Bmatrix}=\begin{Bmatrix}\frac{kr}{kr^2+\underline{K}-I\omega^2}\\1\end{Bmatrix}=\begin{Bmatrix}\frac{-m\omega^2+k}{kr}\\1\end{Bmatrix}}$$

if you were to set $\Theta = 1$ 6/6

-using (2b)

$$(-kr)(1) + (k - m\omega^2)X = 0$$

$$X = \frac{kr}{k - m\omega^2}$$

$$\begin{Bmatrix} \Theta \\ X \end{Bmatrix} = \begin{Bmatrix} 1 \\ \frac{kr}{k - m\omega^2} \end{Bmatrix}$$

-using (3b)

$$(kr^2 + K - I\omega^2)(1) - kr X = 0$$

$$X = \frac{kr^2 + K - I\omega^2}{kr}$$

$$\boxed{\therefore \begin{Bmatrix} \Theta \\ X \end{Bmatrix} = \begin{Bmatrix} 1 \\ \frac{kr}{k - m\omega^2} \end{Bmatrix} = \begin{Bmatrix} 1 \\ \frac{kr^2 + K - I\omega^2}{kr} \end{Bmatrix}}$$

These only give the ratio between amplitudes. So you only need to pick unity value for one or the other ($X$ OR $\Theta$), and one equation (2b OR 3b).